Boyle, R.W. & Froman, D.K.

Velocity of sound in liquids contained intubes. 1929.

VELOCITY OF SOUND IN LIQUIDS CONTAINED IN TUBES.

by

R. W. Boyle and D. K. Froman

Physics Laboratory,
University of Alberta,
April, 1929.

1929
Boyle.

INTRODUCTION

There are points still uncertain concerning the velocity of sound in fluids contained in tubes. The Helmholtz - Kirckhoff theory purports to explain the diminution of velocity almost invariably observed by previous experimenters, and while some researchers claim that this theory sufficiently explains the facts, others are of the opinion that no theory adequately accounts for them all. Most of the experimental work in tubes has been performed on gases, generally air, and in this connection Cornish and Eastman,(1) whose work lends support to the Helmholtz - Kirckhoff theory, recently summarised much of the previous work on this important question.

Relatively little experimental work has been carried out in liquids as the contained fluid; but it may be recalled that Dorsing reported certain cases of increase of velocity in the same liquid when unconfined. Dorsing's(2) method of measurement was that of the Kundt's tube, and unlike most of the experimental work on sound velocities he worked at rather high, though audible, frequencies, about 4000 vibrations per second. These notes he generated by friction, by rubbing metal rods longitudinally with a motor driven friction device. The conclusions he reported, relevent to the work of this paper, are quoted as follows: "(1) The velocity of sound in liquids contained in tubes, contrary to the case for gases,

(1) Phy. Rec. 33, 1929, p.90; 33, 1929, p.258.
(2) Ann. der Physik, 25, 1908, pp.227-251.

INTRODUCTION

There are points still uncertain concerning the velocity of sound in fluids contained in tubes. The Helmholtz - Kirchhoff theory purports to explain the diminution of velocity almost invariably observed by previous observers, and while some researchers claim that this theory sufficiently explains the facts, others are of the opinion that no theory completely accounts for them all. Most of the experimental work in tubes has been performed on gases, especially air, and in this connection Gerlach and [illegible](1), whose work lends support to the Helmholtz - Kirchhoff theory, recently reviewed most of the previous work on this important question.

Relatively little experimental work has been carried out in liquids as the conducting fluid; but it may be recalled that [illegible] reported quite a range of increase of velocity in the [illegible] than [illegible]. [illegible](2) method of [illegible] was [illegible] of the [illegible], and [illegible] of the [illegible] with [illegible] the method of [illegible] high, though [illegible], [illegible], [illegible] have [illegible]. [illegible] by [illegible], by [illegible] [illegible] with a [illegible]. The [illegible], [illegible] to the [illegible] of this paper, [illegible] as follows: "(1) The velocity of sound in [illegible] in tubes, [illegible] of the [illegible].

(1) Phys. Rev. [illegible]
(2) Ann. der Physik, [illegible]

increases with decreasing radius for a given strength of wall, and for a given radius increases for increasing wall strength. (2) When the velocity decreases in tubes the decrease may be ascribed almost exclusively to the elastic forces of the tube and of the liquid; heat conduction and friction are without appreciable influence. (3) Vibrating liquid columns generate sympathetic longitudinal vibrations in the tube wall; hence to obtain Kundt's dust figures it is desirable to make the natural frequencies of the liquid and tube as nearly equal as possible. (4) In vibrating liquids, contrary to the behaviour of gases, the harmonic overtones distinctly assert themselves." In these conclusions are found certain important considerations not comprised in the Helmholtz-Kirckhoff or any similar theory. Busse[3] reverted to Dorsing's method to determine velocities of sound in many liquids for the purpose of determining certain useful thermodynamical constants.

On theoretical considerations of the elasticity of the tubular containing walls there should be corrections applied to the unconfined velocity in a fluid for the waste of energy laterally by the vibrations of the wall. If the walls are not rigid a damping of the vibrations in the contained fluid is caused by the vibration in the walls, which damping causes a diminution of velocity in the contained fluid. This diminution, however, is usually small.

(3) Busse, Ann. der Physik, (4) 75, 1924, pp.657-664.

The fact was observed first by Wertheim (1847) and the first explanation was offered by Helmholtz[4] (1848). Since then the problem has been investigated mathematically by Lamb[5] Green,[6] and others, and most recently by Gronwall.[7] The last undertakes an exact solution of the problem to find the relation between the velocity of sound C in the column of liquid in the pipe or tube and the velocity C_0 in an unlimited body of the liquid. As might be expected the relation is extremely complicated, but by employing suitable approximations the equation may be reduced to $C_0 = \frac{C}{1-\epsilon}$, where C = 2Ln, L being the length of a liquid column under fundamental resonance, and n the resonant frequency of the generating note. ϵ is a complicated function of the inner and outer radii of the tube wall, the chief elastic constants of its material, and the density of the liquid. Pooler[8] found verification of Gronwall's relation experimentally by determining the velocities in a column of liquid contained in a vertical cylindrical steel tube. The column was brought into resonance at an audio frequency by an electromagnetically excited diaphragm at the bottom. When the resonance frequency of the liquid column was the same as that of the diaphragm the reaction of the diaphragm on the system was very small, and the velocity under this condition was easily measured.

4. Helmholtz, Abhandlungen, Vol.1, p.246.
5. Lamb, Proc.Manchester Lit. and Phil.Soc., 42, 1898, No.9. Dynamical Theory of Sound, 1910,ed., p.173.
6. Green, Phil.Mag. 45, 1923, p.907.
7. Gronwall, Phys.Rev. 30, 1927, pp.71-83.
8. Pooler, Phys.Rev. 31, 1928, pp.157-168.

The modern uses of short length ultrasonic waves have introduced new and convenient methods by which the velocity of sound in liquids contained in tubes may be more completely investigated. New and interesting results by this method, as shown in this report, prove that it is easily possible to cause at will largely augmented as well as diminished velocities; that while the factors of wall elasticity and density, and wall thickness, may have a certain importance they are not nearly so important to cause marked changes in velocity as the frequency of vibration and the diameter of the containing tube. While lateral waste of energy in the walls and viscous damping in the liquid itself may be relatively small influencing factors on the velocity in the tube, the dominating factors of all in the ~~determination~~ fixing of tubular velocities are the factors of columnar impedance and of selective absorption. The experiments here described offer abundant proof of this conclusion, but there are theoretical reasons as well which will be discussed in a future paper. Only the initial experimental work will be described here.

Virtually all velocity of sound measurements in liquids contained in tubes, with Dorsing's, Busses', and Hubbard and Loomis'(9) as notable exceptions, have been made at low frequencies, and consequently in tubes whose diameters were small compared with the length of the wave. It is this fact more than any other which has heretofore masked the effect of selective absorption which by the use of ultrasonics may be easily disclosed. Some of the results of this paper support

(9) Phil.Mag. June, 1928, p.1177.

Dorsing's observations, but they offer a different exalenation for some of his observed facts.

METHOD

The primary object of the researches here described was to study by the ultrasonic method the velocity of sound in cylindrical tubes, the carrying medium being a liquid. It was first intended to use tubes of material in which the specific accoustic resistance ρv (where ρ is the density and v the velocity of sound in the material) was less than or comparable with that of the liquid. In such tubes the walls cannot be considered "rigid" in the theoretical sense. During the progress of the work, however, it was found this consideration in comparison with others later described did not greatly matter, and it became expedient to use tubes more nearly "rigid" than at first intended. The frequencies employed in the experiments ranged from about 10,000 to 200,000 cycles per second, and the phase velocities were measured by the method of stationary waves. there arises question whether or not a sound signal at the stated frequencies would travel very long distances in a tube with the corresponding velocities here determined).

The source of ultrasound used in the experiments was a metal rod oscillator, set into longitudinal vibrations by the action of a plate of piezo-electric quartz. The phase velocity in the tube was measured by the length of the standing waves produced by reflection from a reflector in the tube, or, as in later experiments, by two exactly similar oscillators, one at each end of the tube, actuated at the same frequency by parallel connection in the same electric generating circuit. The latter proved to be the better method of experiment.

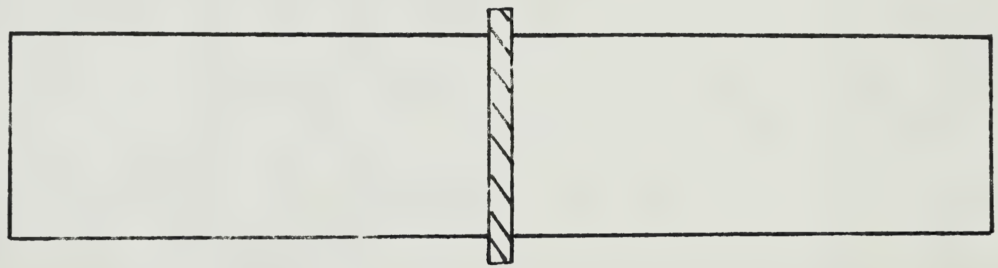
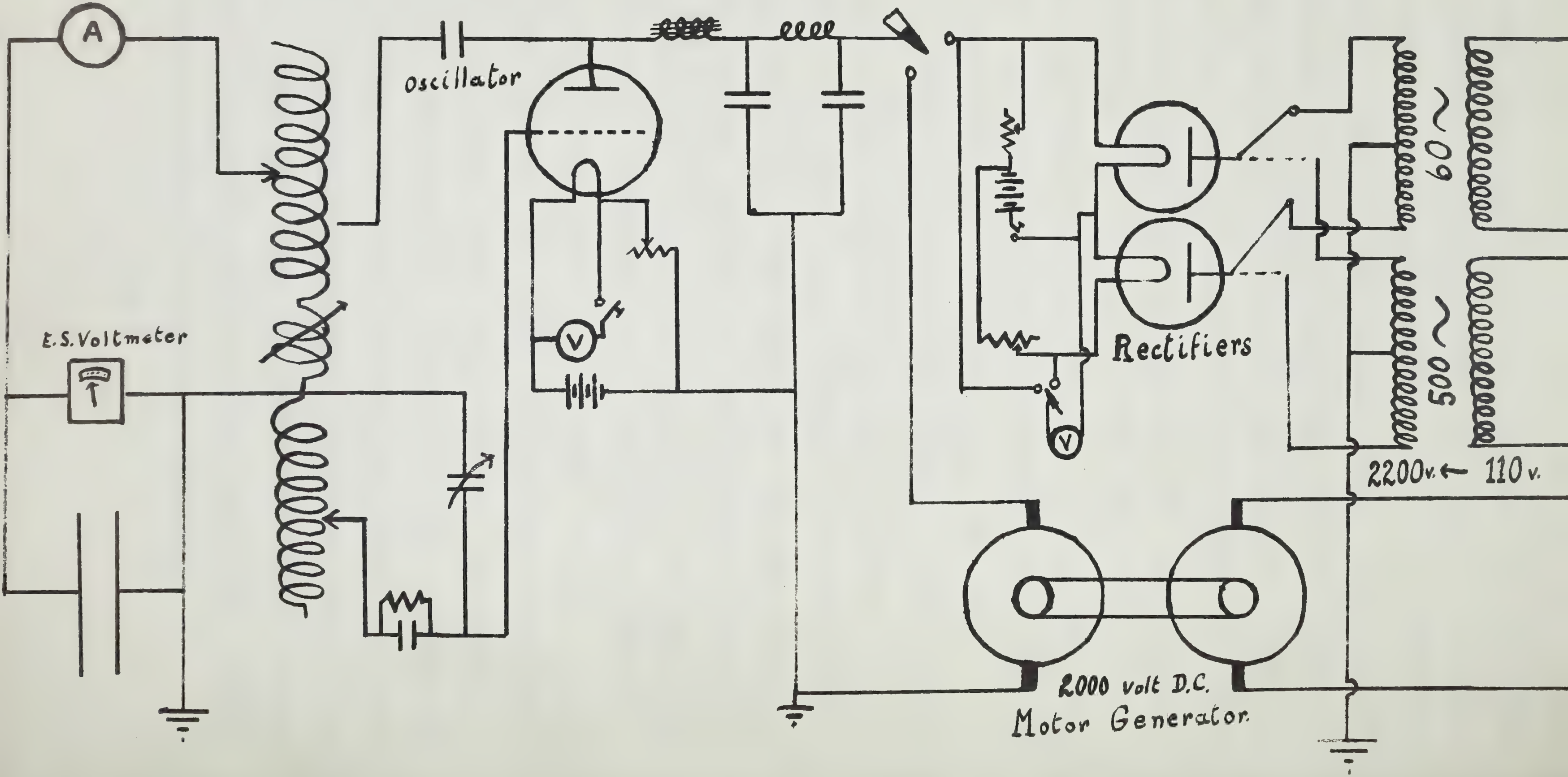


Fig.1.



E.S. Voltmeter
Oscillator
Rectifiers
60 ~
500 ~
2200v. ← 110 v.
2000 volt D.C.
Motor Generator

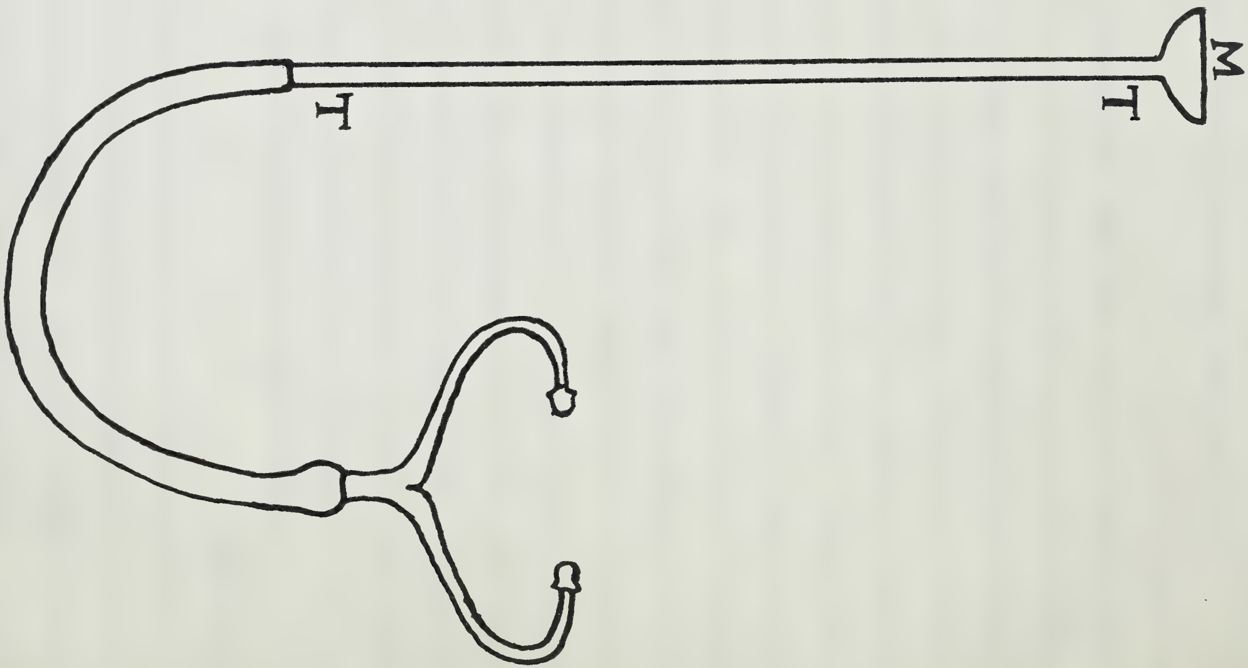
Fig 2

A diagram of an oscillator is given in Fig. I. This type of oscillator consisted of two cylindrical rods of metal usually duralumimum whose diameters were slightly less than the internal diameter of the experimental tube, and whose lengths were long enough to be used at their frequency of fundamental resonance as well as the first few overtones. A thin disc of quartz, cut with its faces perpendicular to an electric axis, was firmly cemented between two exactly similar metal rods so that the compound piece was aligned co-axially. The oscillator thus formed a parallel-plate condenser and as such took its place in the plate circuit of the valve oscillating set shown diagramatically in Fig. II. When appropriate electric power was applied to this oscillating circuit the quartz vibrated by piezo-electric action, and on adjusting the frequency for mechanical resonance of the oscillator the attached rods vibrated with maximum amplitude.

One end of the rod oscillator was fitted snugly inside an end of the experimental tube, all leaking of liquid being prevented by the use of a soft rubber washer or by wax.

It is well known that if the acoustic resistances differ greatly for two media the reflection coefficient, for sound travelling in the one medium and incident on the other, is high. Consequently, as water was first used, it was decided to use an air reflector to create the stationary waves since the acoustic resistances of air and water differ very greatly. The air reflector, Fig. III consisted of a flat, thin, sheet of mica M, fitted over the mouth of a bell-shaped piece of metal at the end of an open tube TT. The bell was of such a size that it fitted snugly into

Fig. 3.



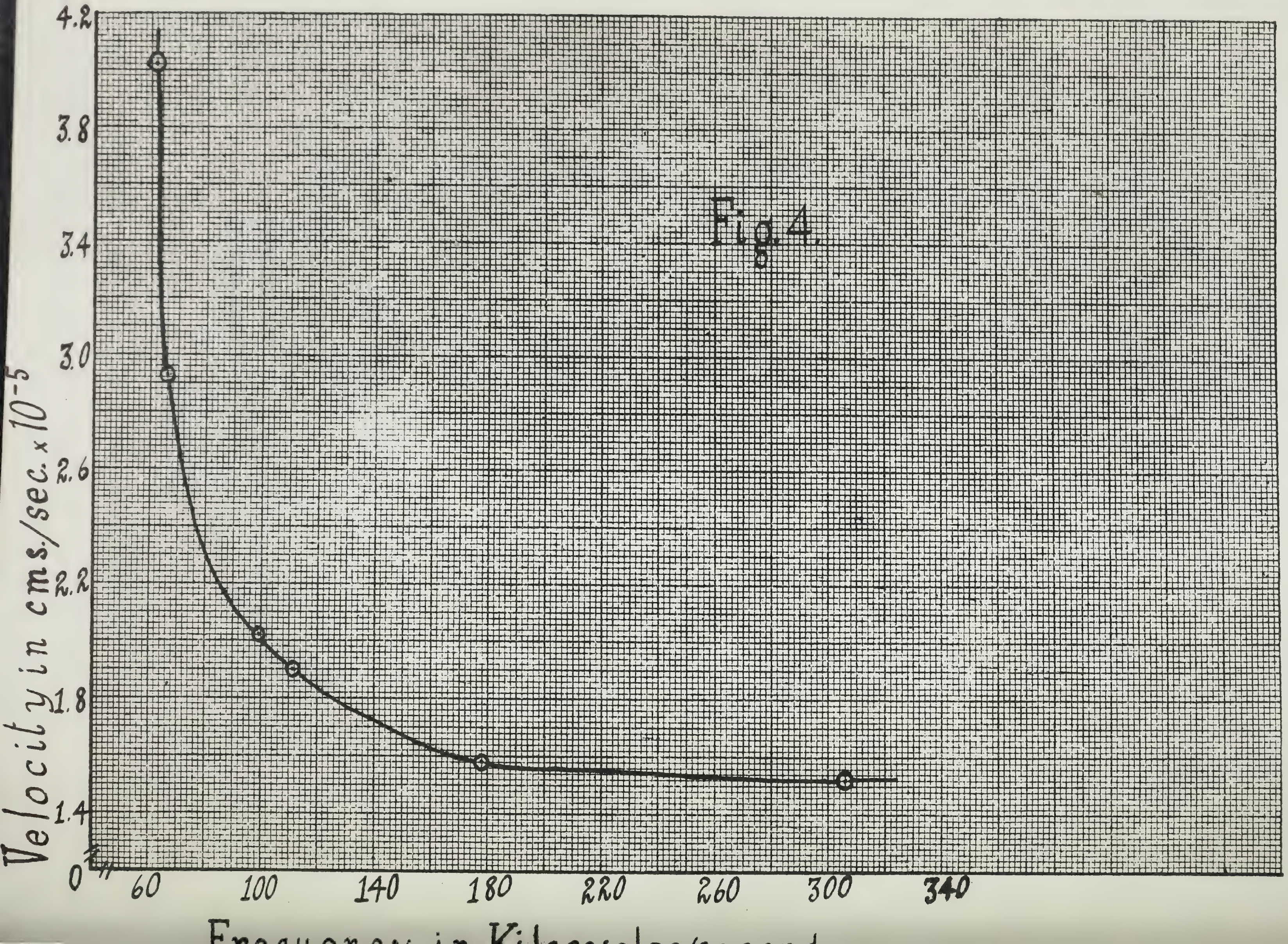
the experimental tube. The electric power applied to the oscillating circuit was rectified 60-cycle alternating current which generated in the oscillating circuit a "tonic-train" of waves. Though the high frequencies employed were usually ultra audible in pitch the 120-cycle note of this tonic train could easily be distinguished by means of a stethoscope attached to the end of the listening tube TT projecting from the bell.

Measurements of the wave-lengths were made by adjusting the position of the reflector in the experimental tube until the sound of the tonic train was a maximum. In this condition the column of liquid between the oscillator and reflector was in resonant vibration, a node of pressure occurring at the mica sheet. The position of the reflector with respect to a measuring scale was noted and then adjusted to the next maximum, which was, of course, one-half wave length distant from the first. This process was carried out throughout the whole length of the tube, or in the case of extra long tubes, until the distance from the source of the waves became so great that the points of detectable maximum amplitude became indistinct. The frequency of the ultrasound was determined by measuring the frequency of the electrical oscillations with a Hertzian wavemeter and the phase velocity was calculated from the velocity, frequency, wave-length relation, $v = n\lambda$.

As mentioned before it was at first intended to employ tubes of small rigidity; consequently the first trials were carried out with tubes of sheet celluloid, 0.04 cms. thick

made by rolling the sheets into a cylindrical form of single thickness and cementing together the overlapping ends. A few tubes of mica, made from mica sheets, were also used, but this material breaks too easily to be rolled into tubes of small diameter. In the first trials the experimental tube was set up vertically the generator being at the bottom end.

At very high frequencies the nodes and antinodes of the stationary waves were found to be very distinct, and the measured phase velocity was the same as the unconfined velocity in a large body of the liquid. A small reduction in velocity might have been possible, but it was soon realized that with a wave length short in comparison with the diameter of the tube the column of liquid acted as an unlimited body. As the wave length was increased, i.e. the frequency diminished, the phase velocity, as measured ~~from~~ by the stationary waves, was found to increase and the distinction with which the nodes and antinodes could be detected became decidedly less. A typical example is given below. The readings taken are given in Table I, and a curve of velocity against frequency is plotted in Fig. 4. At frequencies lower than those quoted standing waves could not be detected in tubes of this diameter, viz. 2 cms. Several curves similar to this one were obtained.



Velocity in cms./sec. x 10^{-5}
4.2
3.8
3.4
3.0
2.6
2.2
1.8
1.4
0
60
100
140
180
220
260
300
340

Fig. 4.

TABLE I

Notes: Tube - celluloid sheet, internal diameter 2.0 cms.
wall thickness about 0.04 cms. Liquid used - water.
Temperature - 18°C.

No. of antinodes observed.	Frequency in cycles per sec.	Wave length in cms.	Velocity in cms./sec.
31	306,000	0.496	1.52×10^5
21	178,000	0.888	1.58 "
10	111,300	1.71	1.90 "
11	99,500	2.03	2.02 "
9	66,000	4.44	2.93 "
4	61,500	6.65	4.02 "

Some time was spent in trying to improve the technique of the experiment, e.g. rolling tubes, making them more truly cylindrical, and in experimenting with new listening devices such, for example, as steel reflectors with small holes in them covered with thin mica. It was thought that irregularities of wall thickness, particular dimensions, or some other intrinsic quality of the experimental tubes might be the cause of the breaking up of the stationary waves and of the inability to detect them at certain frequencies. Rubber and other materials were employed as walls of the experimental tubes, and other curves similar to the one shown in Fig. 4 were so obtained. It was noticed that as the internal diameters of the experimental tubes were increased the velocity-frequency curves retained their shape but the high values of velocity were shifted towards the lower frequencies.

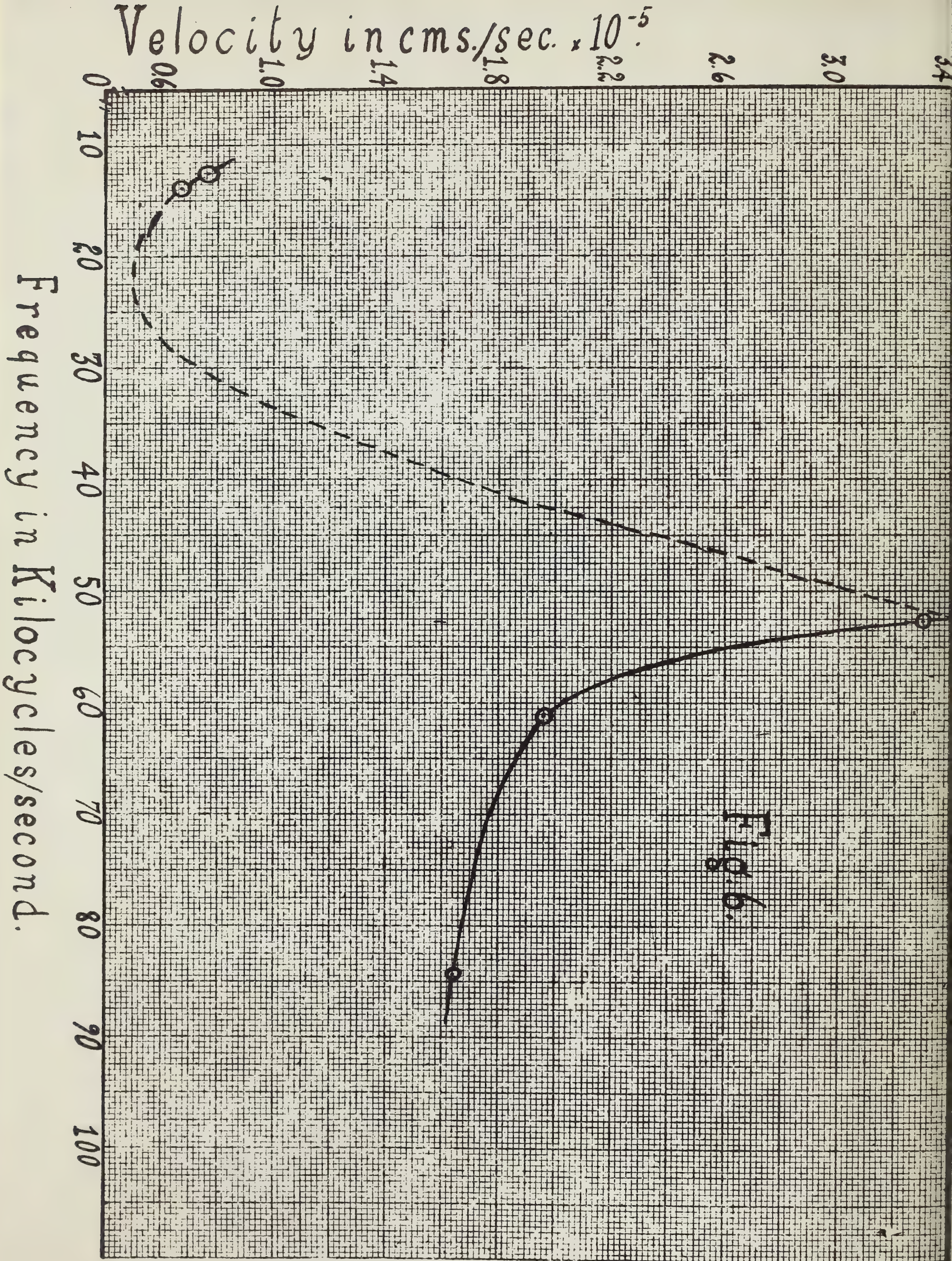

Velocity in cms./sec. x10^-5
0
0.6
1.0
1.4
1.8
2.2
2.6
3.0
3.4
10
20
30
40
50
60
70
80
90
100
Frequency in Kilocycles/second.

Fig. 6.

When a "pyralin" (celluloid) tube, internal diameter 2.0 cms. was used it was found possible to detect stationary waves at lower frequencies with respect to the rise in velocity-frequency curve than were hitherto possible. The readings taken are given in Table II and the velocity-frequency curve plotted in Fig. 6. The important thing to notice is that near the high velocity values there is a gap in the curve which could not be filled in, and that at the frequencies below those corresponding to this region the velocity was very low. Other curves similar to Fig. 6 were obtained, but after many trials it was not found possible to fill in completely the gap in the curve, consequently it was thought that some new method of experiment should be devised.

TABLE II

Notes: Tube - pyralin, internal diameter - 2.0 cms. Wall thickness 0.15 cms. Liquid used - water. Temperature 18°C.

No. of nodes observed	Frequency in cycles per sec.	Wave length in cms.	Velocity in cms/sec.
11	84.400	1.94	1.64×10^5
10	52,800	6.25	3.30 "
4	61,300	3.20	1.96 "
5	12,500	6.10	0.762 "
5	13,900	4.85	0.675 "

Boyle, Lehmann and Morgan (10) some years ago (1923) carried out a few experiments with Kundt's tubes charged with

(10) Trans. Roy.Soc.Can., Vol. XII, Sect.III, 1926, p.371

water charged with cinder dust and placed in an ultrasonic beam projected in a large water tank, but the observed results on the stationary waves formed in the tube were not very definite. Following the same method a quartz block transmitter, of 4 in. diameter radiating face, was set up in a large tank of water, and glass tubes filled with water containing cinder dust were lowered into the emitted ultrasonic beam. Occasionally rather poor figures were formed, but usually nothing particular happened. The results in fact were not as good as those of the former experimenters, although the identical apparatus as used by them was here employed. Some considerable time was spent trying to improve condition, e.g. adjust to most suitable frequencies, and find correct sized tubing for good figures, but the results were so poor that this particular method was abandoned.

A new experiment, the results of which were very illuminating, was next tried. An experimental tube of glass of internal diameter 3.5 cms., lengths varying from 10 to 30 cms. was set up horizontally, and a rod oscillator was fitted into one end. The other extremity of the tube projected through a hole in one end of a small tank of dimensions 60 x 8 x 6cms., and a reflector of metal was placed in the tank at some distance from the mouth of the intruding tube. The tank and tube were filled with water charged with cinder dust. A diagram of the arrangement is shown in Fig. 5. When the ultrasonic oscillator was operated the cinders formed stationary dust figures in the glass tube, and on sifting dust into

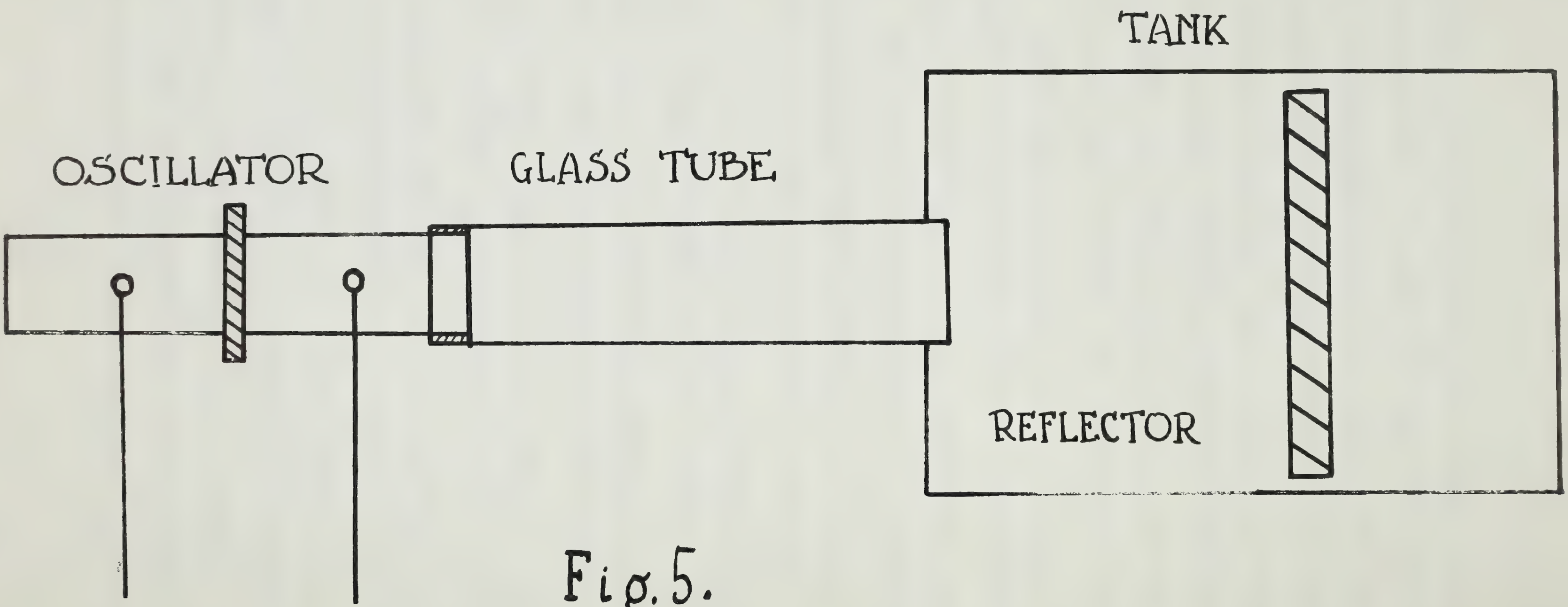


Fig. 5.

the water in the tank in front of the reflector figures of stationary waves were easily disclosed. Measurements of the velocity in the tube and in the tank were taken in this way, and checked by the listener method (page 5) at various frequencies. The resulting velocity-frequency curve for the tube was the same with both dust figure and listener (page 5) methods, and very similar to the one shown in Fig. 4. It is interesting to note that the velocity in the tank remained constant at all frequencies, and that the dust figures would not form in the tube at those frequencies at which stationary waves could not be detected by the listener method. In other words at these frequencies stationary waves did not exist.

A special tank with ends and bottom of wood and sides of celluloid was now constructed. The dimensions were: length 70 cms. height 8cms. and width 3 cms. The end of a rod oscillator of $\frac{3}{8}$ inch diameter intruded through a hole in one end of the tank and a moveable steel reflector could be placed in the tank in any desired position. The liquid in the tank was water. By means of both dust figures and the listening methods the stationary wave field was mapped out. The resulting velocity-frequency curve was the same by both methods and was very much like that of Fig. 4. In this case the cross-section of the liquid column was not circular nor was the liquid completely surrounded with wall or otherwise enclosed, which showed that the variation of velocity with frequency was not dependent on the sectional form of the

liquid column, or on its partial or complete enclosure by a wall. Thus in the course of this research it was easily grasped that increased or diminished phase velocities could be obtained at will, and it became necessary to investigate what were the influencing factors of the velocity changes by more precise methods.

In experimental methods based on stationary waves it is always preferable to be able to <u>see</u> the representations of the nodes and antinodes from which the measurements are taken. Consequently in the further course of this research it was decided to employ <u>transparent</u> experimental tubes. Moreover (11) by taking advantage of the phenomenon of ultrasonic cavitation bubbles of gas produced in the liquid by the ultrasonic vibrations could be used to mark out the nodes of stationary waves, and thus avoid the necessity of charging the experimental liquid with dust or other impurities of any kind.

At any given pressure in a liquid containing dissolved gas if stationary waves exist and are sufficiently energetic, small bubbles of gas will form throughout the liquid and be driven to the nodes of displacement by the pressure of radiation of the waves. Therefore, if an ultrasonic beam be directed vertically upwards through a liquid column, layers of these bubbles will be formed, one-half wave length apart, parallel to a horizontal reflecting surface; if the beam and column are horizontal the bubbles will be driven to the nodes and rise in vertical curtains, one half wave length apart, in the nodal planes.

(11) Boyle, Science Progress, XXIII, July, 1928, p.94.

Naphtha is a liquid easily made to bubble,[11a] consequently it was decided to use it as the experimental liquid enclosed in glass or pyralin tubes. The tubes were set up vertically and the length of naphtha column could be adjusted. Stationaries were produced by reflection at the free air-liquid surface, and the wave lengths measured by the distance between parallel layers of bubbles.[11b] The results of a typical experiment are given in Table III and in Fig. 7 the velocity-frequency curve is plotted. It will be noticed that the gap shown in previous velocity-frequency curves is now fairly well filled in, but it is important to observe that only a very few regular nodes could be detected at frequencies near the peak of the curve. The formation of stationaries at frequencies appropriate to this peak was very poor; the nodes were both few and irregular, and the indicating gas bubbles shoot along the tube in the direction away from the oscillator as if almost all the energy were travelling in that direction, very little being reflected directly back.

(11a) Boyle, Taylor and Froman. Trans. Roy. Soc. Can. Sect. III May 1929.

(11b) Nature, Oct. 1, 1927. p.476

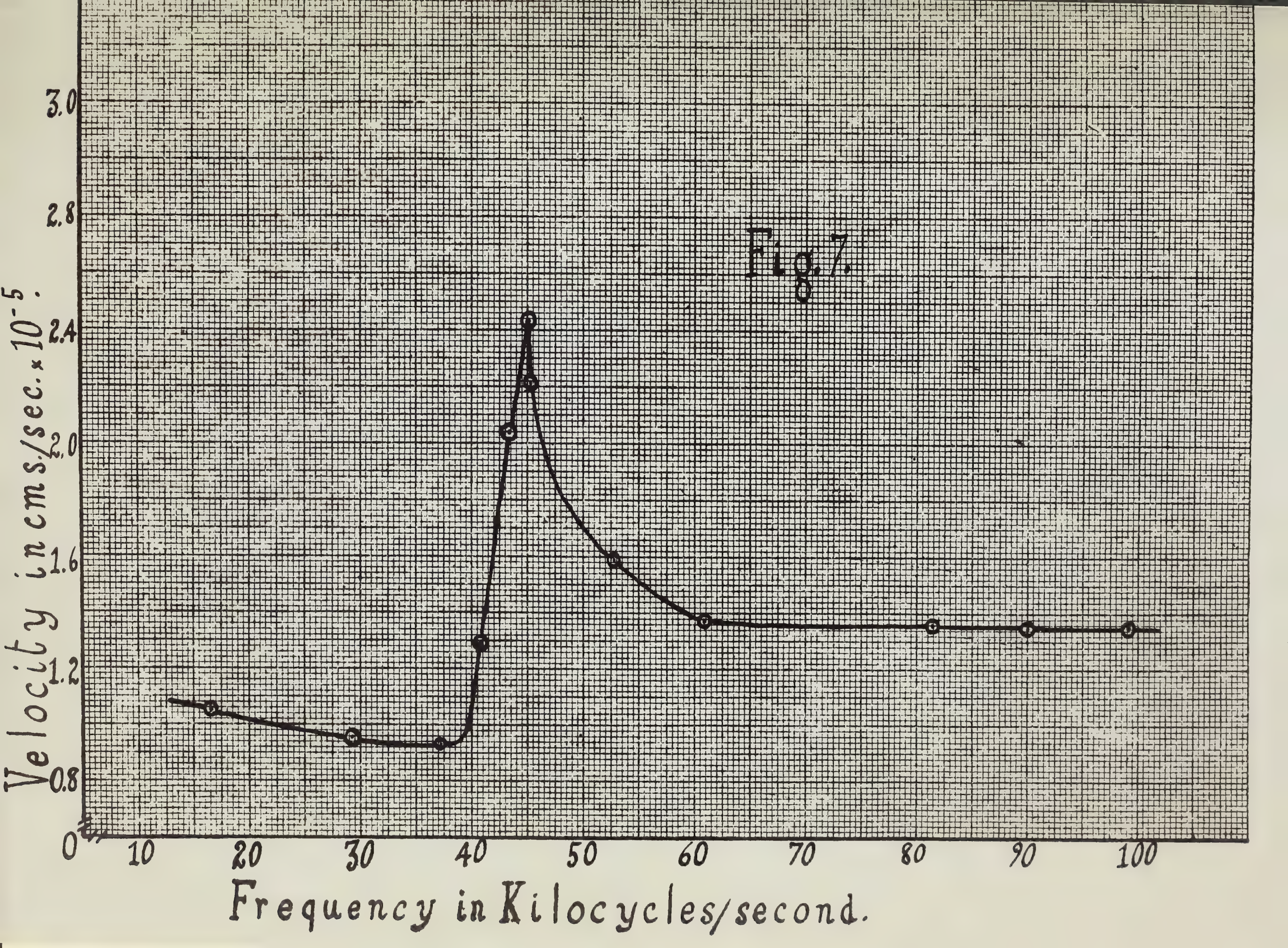


Fig. 7.
Velocity in cms/sec. × 10⁻⁵
3.0
2.8
2.4
2.0
1.6
1.2
0.8
0
10
20
30
40
50
60
70
80
90
100
Frequency in Kilocycles/second.

 Table III

No. of nodes observed.	Frequency in cycles per sec.	Wave length in cms.	Velocity in cms./sec.
4	52,900	3.00	1.59 x 10^5
6	60,800	3.25	1.37 "
4	81,450	2.50	1.36 "
8	99,100	1.36	1.35 "
5	40,900	3.17	1.29 "
3	45,250	4.90	2.21 "
7	29,300	3.27	0.959 "
5	16,550	6.35	1.05 "
3	43,450	4.70	2.04 "
4	45,010	5.40	2.43 "
7	90,000	1.50	1.35 "
6	37,200	2.48	0.923 "

The curve Fig. 7 bears a resemblance to the well known selective dispersion ("anomalous" dispersion) curve of optics, where the index of refraction (which is proportional to the reciprocal of the phase velocity) is plotted on a frequency base. Here the velocity itself is plotted, but the characteristic fall, sharp rise, and subsequent fall of the velocity frequency curve are unmistakeable. Selective dispersion is caused by the selective absorption of energy at the frequency of the sharp discontinuity of the velocity-frequency (or reciprocal of velocity and frequency) curve. It thus appears that in the present case there is a special absorption of energy at the frequencies of and near the peak of the velocity-frequency curve of Fig. 7. Under such

conditions of absorption it would be difficult for stationary waves to form in the tube. Rather there would be a composite of stationary wave and wave transmission characteristics, the standing wave pattern tending to disappear and the wave transmission becoming the more pronounced as the absorption of energy increased. The greater the absorption the more the vibrational impedance of the liquid column becomes like a pure resistance having no reactance components. When the impedance of the column is relatively without resistance and consists mostly of inertia reactance the standing wave pattern will be most pronounced.[(12)] Thus the problem of this research opened out into an investigation to determine how the energy became absorbed, and how the frequency for this selective absorption depended on the dimensions of the column of liquid, of the tube walls, and on the materials of both. The research has not been concluded, but the remainder of this report will contain the results obtained thus far.

An improved method of experiment was devised. By using two transmitters one at each end of the experimental tube, to form the stationary waves instead of relying on the reflection by a reflector at one end only it was found that better stationaries could be produced, especially at the troublesome frequencies at and near the peak of the velocity frequency curve. Identical transmitters, which were connected

(12) Crandall - Theory of Vibrating Systems and Sound, ed. 1926 pp.100-103.

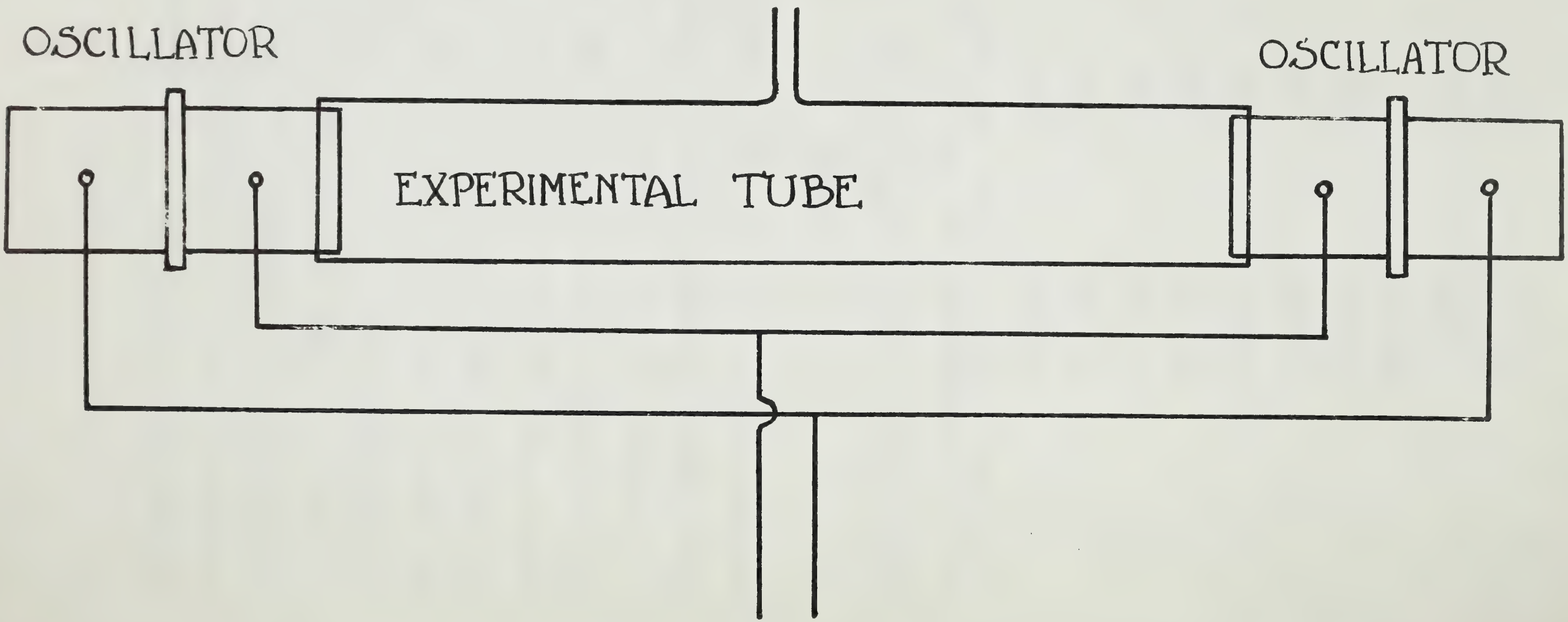


Fig. 8.

in parallel to the oscillating electrical circuit, were fitted into each end of an experimental tube. This tube was provided with a branch pipe near its middle through which the tube could be filled with the experimental liquid, and to which a vacuum pump could be attached to facilitate cavitation in the liquid by lessening the internal pressure. The arrangement was as shown in Fig. 8. The distance between the transmitters was varied, but with little effect on the stationary waves except when the tube was long enough to cause a significant loss of intensity at the centre. The result of one set of readings, taken for naphtha in a glass tube of diameter 3.5 cms. are given in Table IV.

TABLE IV

Notes: Tube - Glass; Length of tube - 57.2 cms. Internal diameter - 3.50 cms. Wall thickness - 0.2 cms.

No. of nodes observed.	Frequency in cycles per sec.	Wave length in cms.	Velocity in cms./sec.
12	29,000	3.20	0.93×10^5
8	53,200	2.66	1.41 "
8	34,600	2.61	0.932 "
6	45,100	3.44	1.55 "
6	25,700	3.84	0.987 "
5	73,900	1.82	1.35 "
3	39,500	5.30	2.09 "
3	34,375	2.70	0.928 "
3	34,900	2.60	0.907 "
2	37,700	5.40	1.66 "

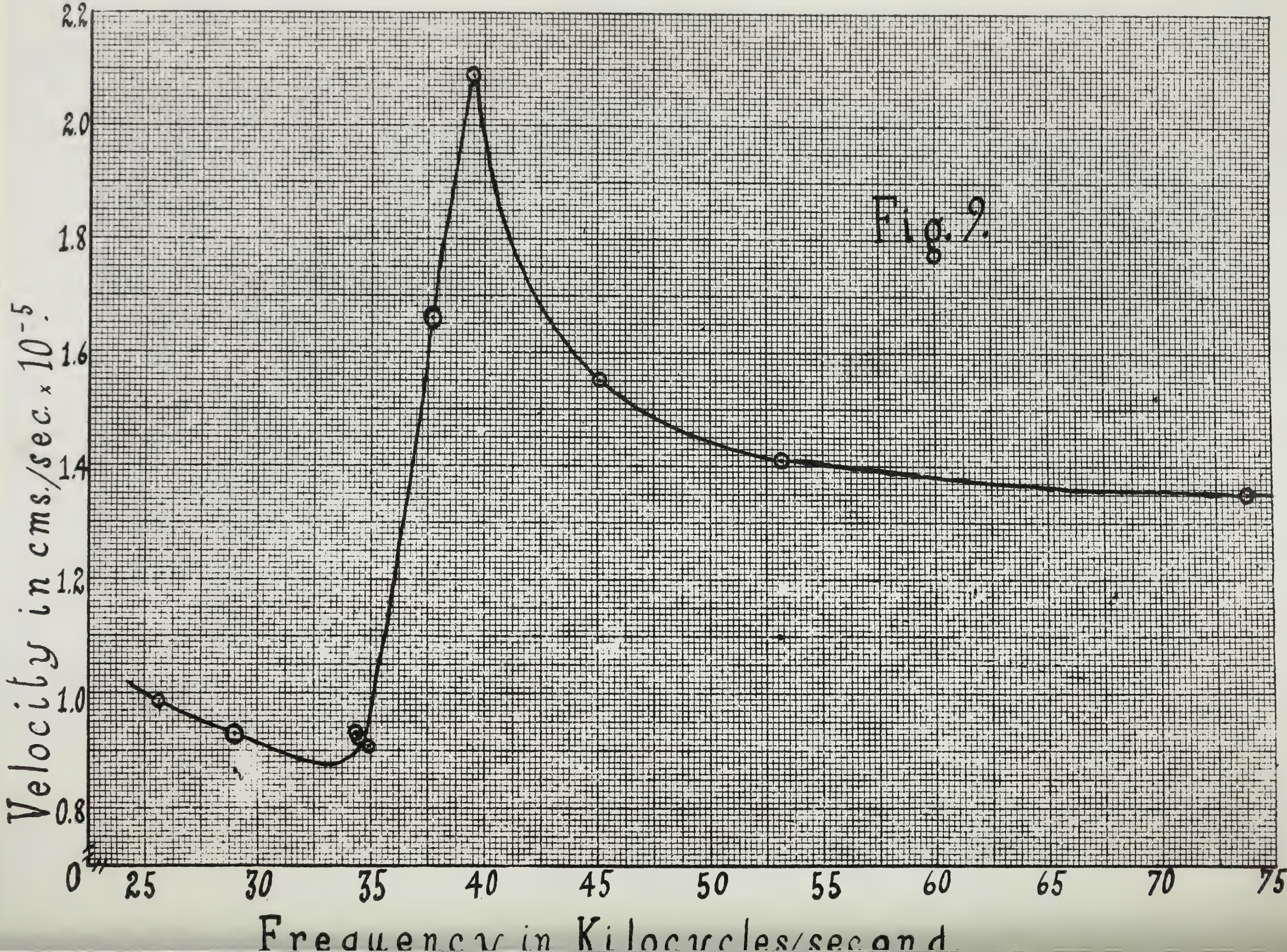

Velocity in cms./sec. x 10^-5
0
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
25
30
35
40
45
50
55
60
65
70
75
Frequency in Kilocycles/second

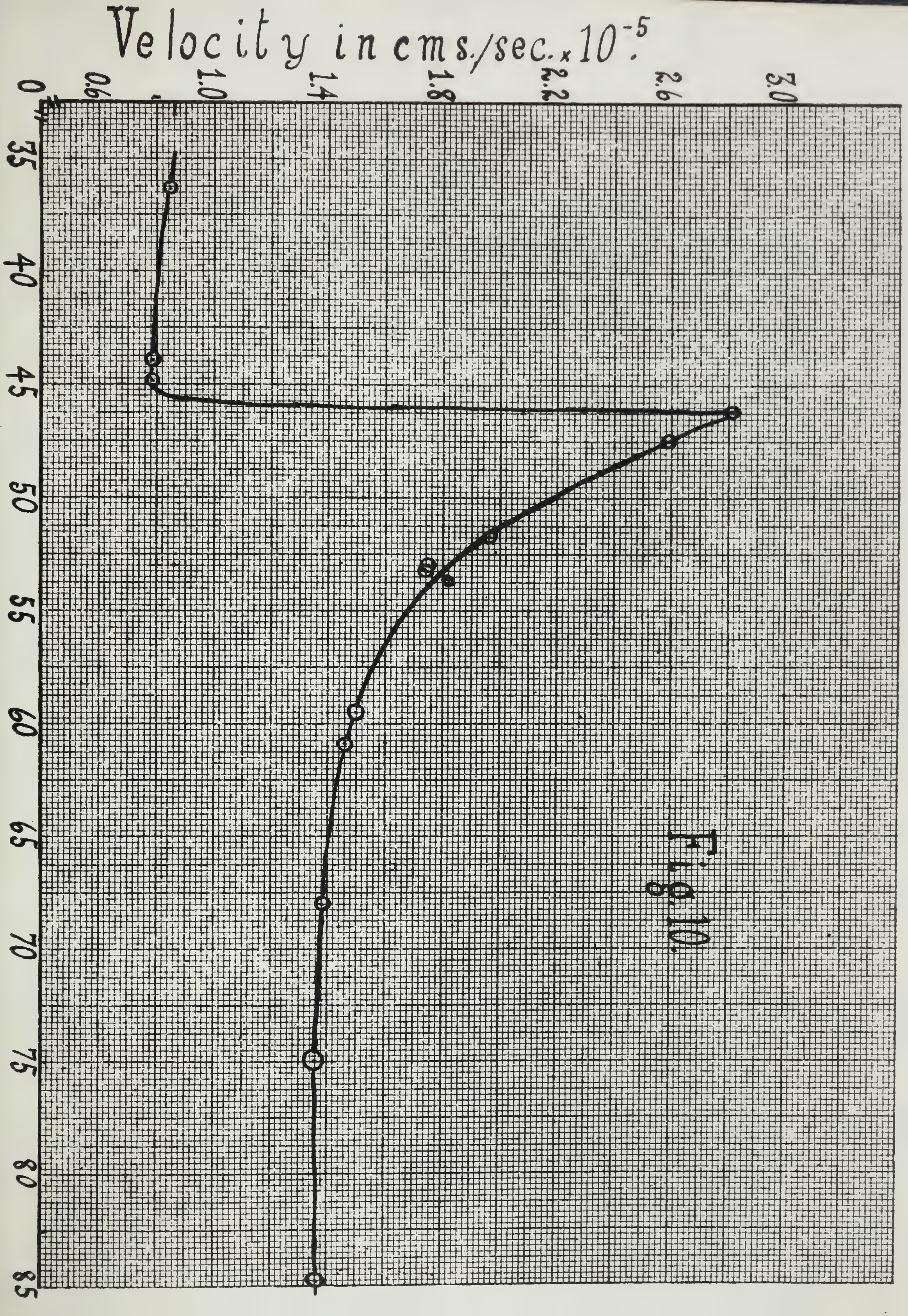
Fig. 9.

The velocity frequency curve is plotted in Fig. 9. The results using the same liquid and a pyralin tube of diameter 1.9 cms. are given in Table V, and the corresponding curve plotted in Fig. 10.

TABLE V

No. of nodes observed.	Frequency in cycles per sec.	Wave length in cms.	Velocity in cms., sec.
11	75,000	1.81	1.36×10^5
11	84,800	1.60	1.36 "
6	68,000	2.02	1.39 "
3	53,000	3.30	1.75 "
5	44,900	1.75	0.786 "
4	60,900	2.40	1.46 "
5	53,200	3.27	1.74 "
4	59,500	2.52	1.50 "
5	53,700	3.40	1.82 "
7	46,150	6.10	2.81 "
5	51,700	3.80	1.96 "
5	47,500	5.45	2.59 "
5	43,950	1.79	0.787 "
6	36,300	2.33	0.845 "

That this method of experiment was a great improvement on the former methods could be seen in the fact that more and more regular stationary waves could be detected at the difficult frequencies in the vicinity of the peak of the velocity-frequency curve.



Velocity in cms./sec.x10^{-5}
0
0.6
1.0
1.4
1.8
2.2
2.6
3.0
35
40
45
50
55
60
65
70
75
80
85

Fig. 10.

TABLE VI

No.of nodes observed	Frequency in cycles, sec.	Wave length in cms.	Velocity in cms/sec	Length of tube in cms.	Mark on curve
7	45,300	3.33	1.51×10^5	55.7	
4	37,350	6.33	2.36 "	55.7	Y
7	30,700	4.92	1.89 "	55.7	
5	38,400	5.07	1.95 "	54.4	
5	41,200	4.20	1.73 "	54.4	⊗
3	37,350	6.30	2.35 "	54.4	
3	37,400	6.05	2.26 "	52.1	
11	44,900	3.32	1.49 "	52.1	
3	37,050	5.90	2.18 "	52.1	⊙
6	45,100	3.36	1.51 "	52.1	
4	38,600	4.93	1.90 "	52.1	
3	38,400	5.05	1.94 "	50.0	
11	45,000	3.34	1.50 "	50.0	×
3	37,300	6.07	2.26 "	50.0	
17	45,200	3.39	1.53 "	41.1	ϕ
4	37,400	5.94	2.22 "	41.1	
3	37,050	6.00	2.22 "	29.9	⊕
11	45,000	3.30	1.48 "	29.9	

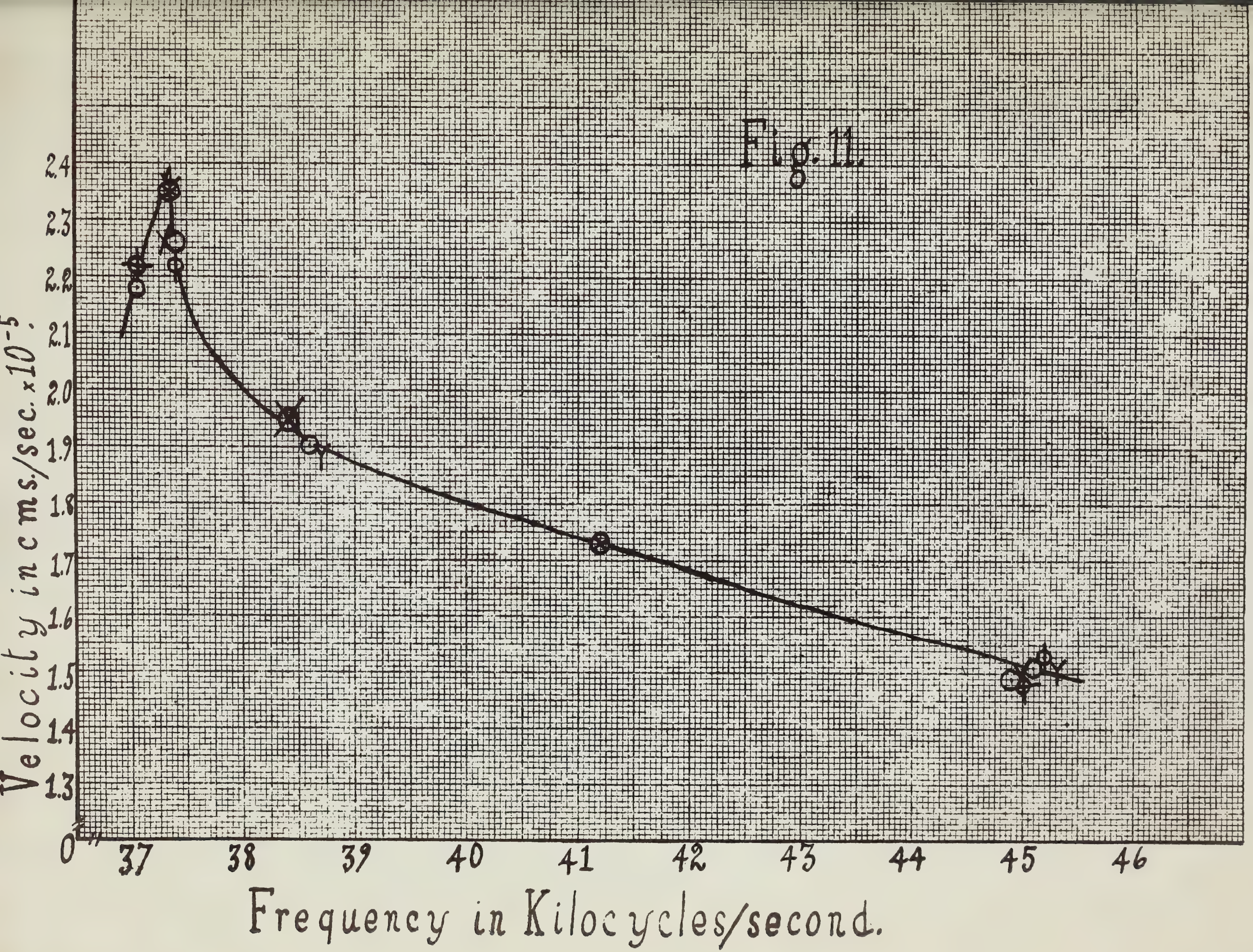


Fig. 11.
Velocity in cms./sec. x10^-5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
0
37
38
39
40
41
42
43
44
45
46
Frequency in Kilocycles/second.

Influence of the Length of the Tube Wall

To determine whether longitudinal resonance in the tube wall as claimed by Dorsing was the cause of the absorption special experiments were begun with a long piece of glass tubing. This tube was shortened many times and the velocity frequency curve for the containing liquid was retaken at each length. The shortening was by amounts of about 1/8 wave length of longitudinal wave in the wall, at the peak frequency of the velocity curve, for a range over a wave length, and after that at irregular intervals. The results are given in Table VI and the corresponding velocity-frequency curve plotted in Fig. 11. It is important to note that this curve remained exactly the same for all lengths of tube experimented with. Herein is a difference from the observations of Dorsing, referred to (p.2) viz., that the vibrating liquid columns generate sympathetic longitudinal vibrations in the tube wall, and to obtain Kundt's figures one must make the natural frequencies of liquid and tube as nearly equal as possible. It made no difference here to the formation of the stationary waves, as indicated by the bubbles in the nodal planes, how long was the length of the experimental tube. In the first experiment of this kind the rod oscillator was not large enough in diameter to fill completely the end of the experimental tube and a thick washer of soft rubber had to be inserted to

fill the annular space between oscillator and tube, in which case possibly the vibrations from the oscillator were not as well communicated to the tube walls. But a special experiment was later performed with a glass tube, internal diameter 3.1 cms. using oscillators which fitted very snugly into the ends of the tube and driven with maximum power at their fundamental resonant frequency, which frequency was made about the same as the frequency of the peak of the velocity-frequency curve. Under such conditions vibrations must have been easily communicated to the tube walls, but there were no noticeable differences in the experimental results. The experimental liquid in this particular case was transformer oil.

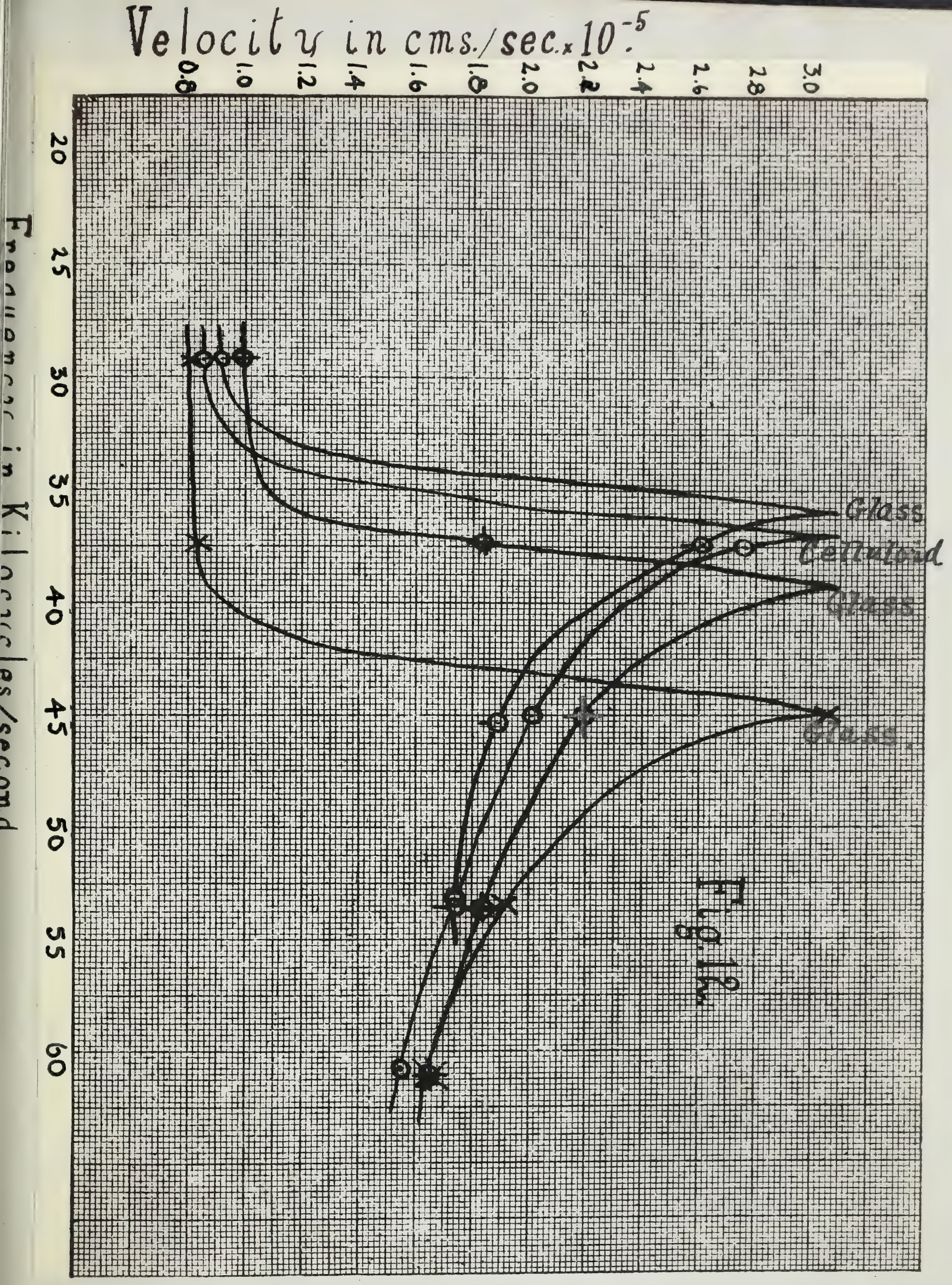
TABLE VII

No. of nodes observed	frequency in cycles per sec.	Wave length in cms.	Velocity in cms./sec.	Length of tube in cms.	
7	47,800	4.10	1.96×10^5	27.45	Internal diameter 3.1 cms.
5	46,800	4.29	2.01 "	26.5	
5	46,800	4.30	2.015 "	25.4	Diameter of pistons 2.8 cms.
2	46,800	4.30	2.015 "	16.7	
2	46,800	4.30	2.015 "	15.5	
2	46,800	4.30	2.015 "	14.5	
2	46,800	4.30	2.015 "	13.5	
2	46,800	4.30	2.015 "	12.4	

any shift in the position of the peak of the curve along the frequency base would have been shown by large differences in the value of the velocity, since the curve is very steep at and near this particular frequency. Table VII gives the values of the velocity obtained for different lengths. They fit within experimental error on the curve shown in Fig. 11. If there were any shift of the peak for different lengths of tube it was very small indeed, and could not account for the large differences in peak velocities noticed for tubes of different diameters.

Different Materials as Experimental Liquids.

Castor oil and water as experimental liquids were now employed. They were made to bubble and form stationary waves by reducing the internal pressure. The results were of the same kind as those for naphtha, but the peak of the curve occurred at a different frequency for each liquid used. Transformer oil was found to be the most convenient liquid in all these experiments, because the bubbles produced by cavitation were finer and rose through the liquid more slowly owing to the greater viscosity.



Velocity in cms./sec.x10^{-5}
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
Frequency in Kilocycles/second
20
25
30
35
40
45
50
55
60
Glass
Celluloid
Glass
Glass.
Fig. 12

Influence of Wall Thickness.

Special experiments were performed to find what effect, if any, the thickness of the wall of the tube had upon the peak frequency of the velocity curve. Up to this time it had not been possible to obtain a number of tubes of the same wall material having exactly the same internal diameter but different wall thicknesses. Fortunately four glass tubes varying only slightly in internal diameter but with large differences in wall thickness were now found. The velocities at various frequencies using transformer oil in these tubes were measured. The lengths of the tubes were different, but it had been shown previously that the velocity and peak frequency of the velocity-frequency curve were independent of the length. The results of these measurements are given in Table VIII, and the corresponding velocity -frequency curves are plotted on the same frequency base in Fig. 12. It is important to observe from the curves that the peak occurs at higher frequencies for smaller diameters, but is not displaced regularly with respect to wall thickness.

Afterward it became possible to obtain four more glass tubes of the same internal diameters, viz. .310 cms. but of different wall thicknesses. The readings for the velocities in the transformer oil at frequencies near that of the peak are given in Table IX, and all velocities are plotted on the same frequency base in Fig.13. It is easily seen from the curve that the thickness of the wall of the

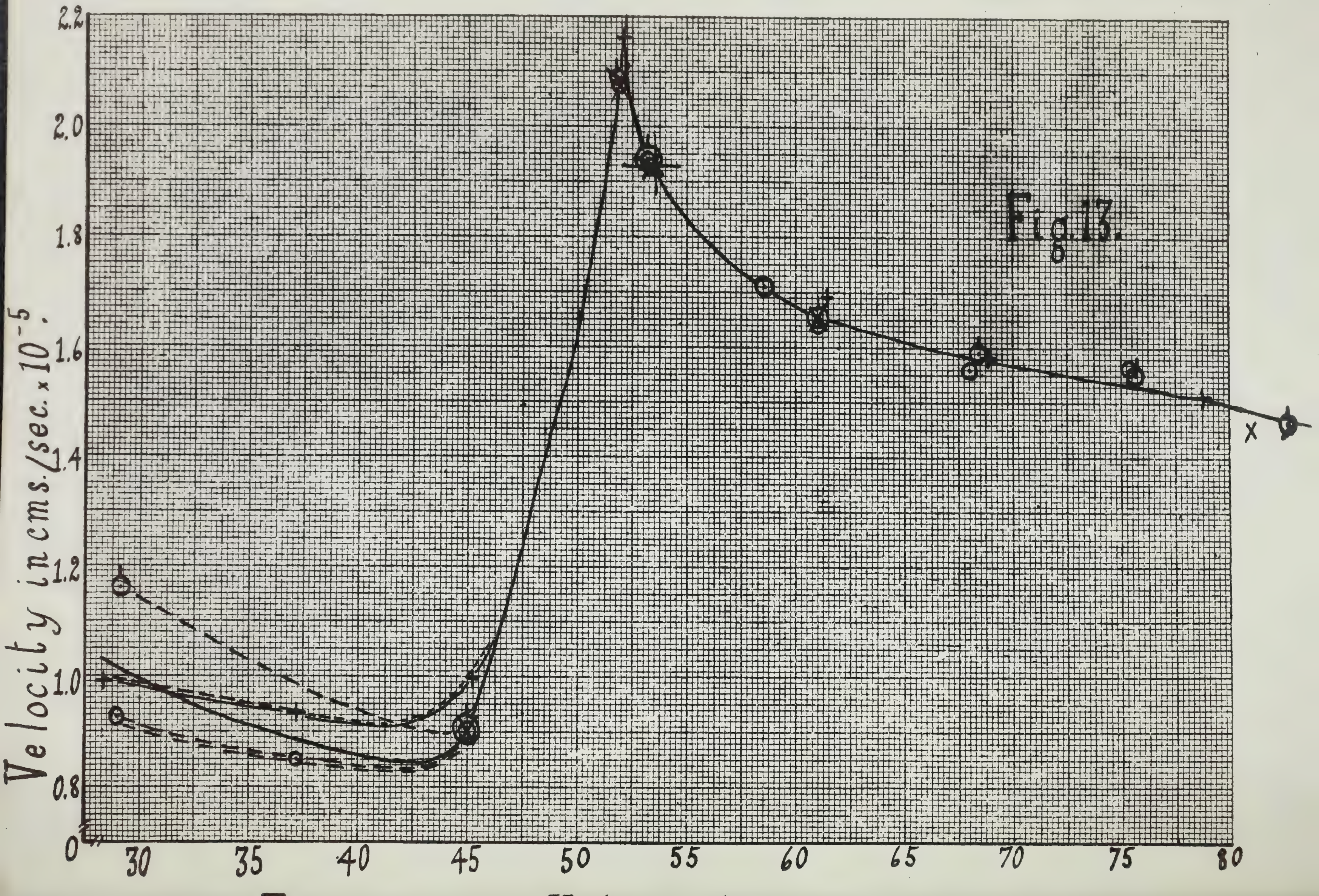

Velocity in cms./sec. x 10^-5
0
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
30
35
40
45
50
55
60
65
70
75
80

Fig. 13.

TABLE VIII

No. of nodes observed	Frequency in cycles, sec.	Wave length in cms.	Velocity in cms., sec.	Tubing.	Mark on Curve
4	53,000	3.30	1.75×10^5	Celluloid	
2	37,300	7.40	2.76 "	length 25.0 cms.	
2	44,800	4.50	2.02 "		
3	60,700	2.58	1.56 "	internal diameter 3.35 cms.	
3	28,980	2.95	0.855 "	wall 0.04 cms.	
2	53,300	1.75	1.87 "	Glass	
2	44,800	4.90	2.20 "	length 37.0 cms.	
2	37,250	4.90	1.8[illegible] "	internal diameter	
2	29,000	3.36	0.975 "	3.20 cms.	
5	60,900	2.72	1.66 "	wall -0.35 cms.	
5	60,900	2.72	1.66 "	Glass	
3	53,250	3.60	1.92 "	length 36.8 cms.	
2	44,800	5.60	3.07 "	internal diameter	X
3	29,050	2.80	0.814 "	2.05 cms.	
4	37,250	2.20	0.820 "	wall ~~[illegible]~~ 0.10 cms.	
8	53,250	3.14	1.67 "	Glass	
6	45,100	4.20	1.90 "	length 15.0 cms.	
2	37,300	7.00	2.61 "	Internal diameter	
7	29,000	3.17	0.920 "	3.5 cms. wall 0.20 cms.	

TABLE II

No. of nodes observed	frequency in cycles, sec.	Wave length in cms.	Velocity in cms, sec.	Wall thickness in cms.	Mark on curve.
4	28,950	3.19	0.924×10^5	0.12	
3	37,200	2.28	0.85 "	"	
2	45,000	2.00	0.90 "	"	
4	75,400	2.07	1.56 "	"	⊙
3	68,000	2.29	1.56 "	"	
3	60,800	2.70	1.64 "	"	
5	58,650	2.92	1.71 "	"	
2	51,900	4.00	2.076 "	"	
2	53,200	3.64	1.94 "	"	
5	53,300	3.62	1.93 "	0.10	
2	45,000	2.00	0.90 "	"	
5	61,000	2.70	1.65 "	"	×
2	51,900	4.00	2.076 "	"	
3	80,800	1.80	1.45 "	"	
3	51,800	4.04	2.09 "	0.25	
2	53,250	3.64	1.94 "	"	
2	45,000	2.00	0.90 "	"	
4	66,400	2.38	1.59 "	"	
2	61,000	2.70	1.65 "	"	
3	81,400	1.80	1.46 "	"	
3	75,700	2.05	1.55 "	"	
3	29,100	4.00	1.16 "	"	
5	61,400	2.75	1.69 "	0.20	
5	45,250	2.20	0.996 "	"	
2	53,500	3.60	1.93 "	"	+
3	37,200	2.50	0.935 "	"	
3	77,500	1.95	1.51 "	"	
3	68,900	2.80	1.56 "	"	
3	52,100	4.15	2.16 "	"	
5	28,300	3.50	0.99 "	"	

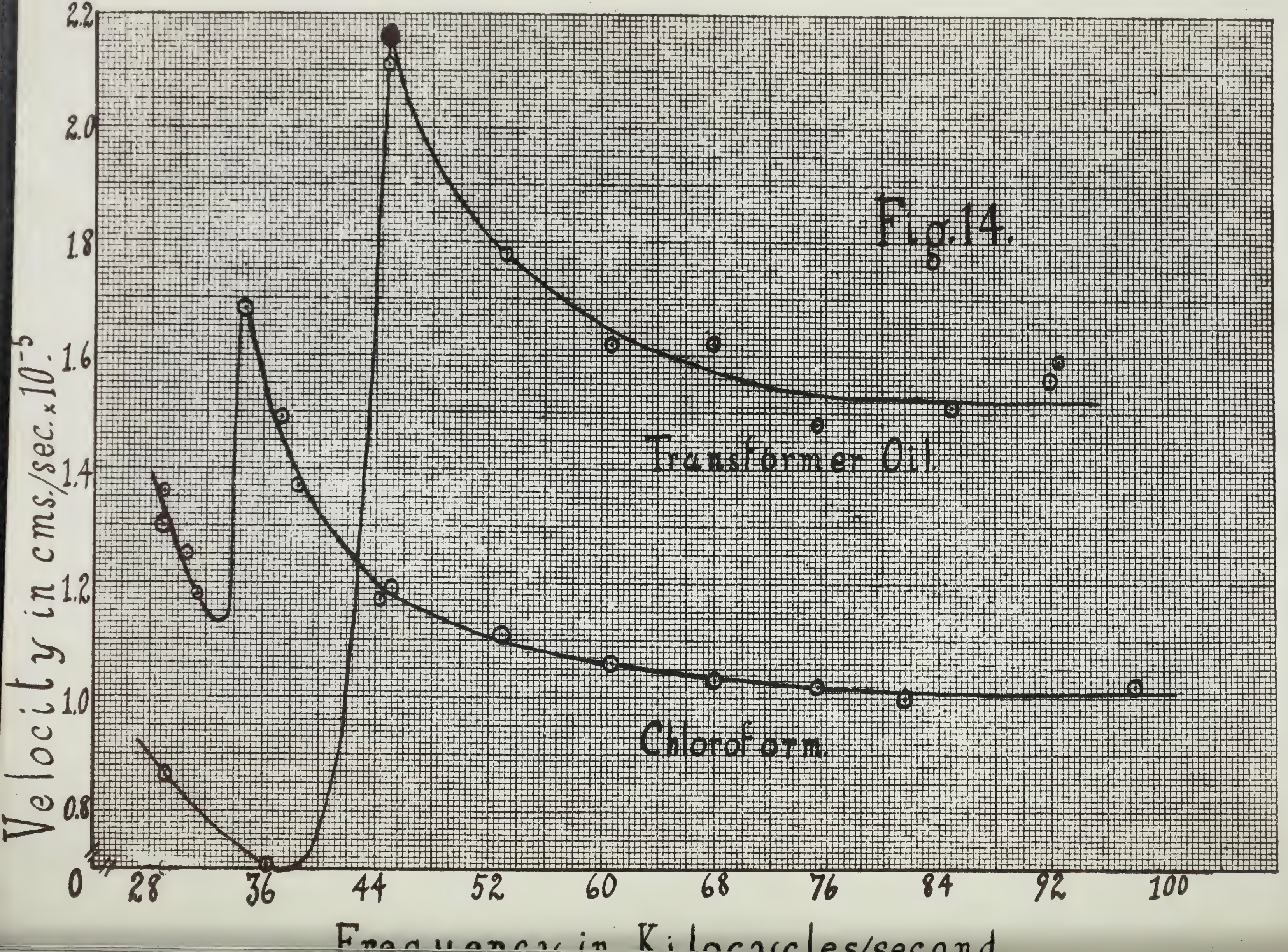


Fig.14.
Velocity in cms./sec. x 10^-5
0
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
28
36
44
52
60
68
76
84
92
100
Frequency in Kilocycles/second
Transformer Oil
Chloroform.

tube does not produce any appreciable effect on the corresponding velocities or on the value of the peak frequency, in comparison with the great effects caused by changing the diameter of the tube.

Influence of Propagating Liquid

It had been noticed that for the same tube the position of this peak depended on the contained liquid. Special experiments to investigate this point were now carried out, using chloroform and transformer oil as liquids under the same experimental conditions and in the same tube. The results are tabulated in Table X, and the velocities are plotted on the same frequency base in Fig. 14. The peak is definitely at a different frequency for each liquid, and as can be seen from the readings at the higher frequencies, the normal "unconfined" velocities of sound in chloroform and transformer oil are quite different and agree with those determined by other methods.

In order to find the effect, if any, on the change of velocity of the energy intensity in the generating waves, the intensity of the ultrasound emitted from the oscillators was ~~changed~~ varied greatly on several occasions; but the results for different intensities were always the same. On some occasions the applied high frequency voltage on the transmitters was doubled, which increased the ultrasonic energy intensity fourfold, but without any noticeable change in the measured velocity.

TABLE [illegible]

No. of nodes observed	Frequency in cycles/sec	Wave length in cms.	Velocity in cms./sec.	Notes.
2	28,950	4.50	1.30×10^5	Tube-glass Internal diameter 3.1 cms.
4	37,350	3.98	1.49 "	
11	53,000	2.10	1.11 "	Wall 0.10 cms.
11	60,750	1.75	1.06 "	Liquid Chloroform
3	38,550	3.58	1.37 "	
5	30,600	4.07	1.25 "	
3	31,200	3.79	1.18 "	
3	34,600	4.85	1.68 "	
5	44,350	2.64	1.17 "	
5	45,175	2.64	1.19 "	
5	29,000	4.69	1.36 "	
11	75,150	1.36	1.02 "	
11	81,600	1.22	1.00 "	
11	68,000	1.51	1.03 "	
11	98,000	1.11	1.02 "	
7	29,070	2.97	0.864 "	Liquid Transformer oil.
7	53,250	3.34	1.78 "	Tube same as above.
3	44,900	4.70	2.11 "	
3	45,000	4.80	2.16 "	
2	44,900	4.80	2.16 "	
7	37,250	1.90	0.707 "	
7	60,700	2.67	1.62 "	
6	92,000	1.70	1.56 "	
7	84,900	1.78	1.51 "	
5	92,500	1.72	1.59 "	
7	75,400	1.97	1.48 "	
3	68,000	2.38	1.62 "	

An attempt was also made to find other peaks ("absorption bands") in the velocity-frequency curve under any given conditions. It was thought that if there were more than one peak in the curve they might occur at harmonics of the frequency of the first, since the effect shown here is no doubt due to selective resonance of oscillation in some possible degree of freedom. In order to employ waves of sufficient length to make accurate measurements at the higher harmonic frequencies, it was decided to ~~use~~ start with a long wave length a tube of large diameter. A tube of internal diameter 7.9 cms. was filled with transformer oil, and observations of the velocity were taken for frequencies ranging from 15,000 to 100,000 cycles per second. The results are given in Table XI and the velocity-frequency curve plotted in Fig. XV. It will be noticed that there is the selective absorption peak at a frequency of 25,000 c/sec. and an indication only of a second peak between 45,000 and 50,000 c/sec. but this is not certain. The curve is quite irregular between 40,000 and 70,000 and difficult to interpret. After 70,000 it seems to steady down to the normal velocity. With tubes of much smaller diameters the first selective absorption peak occurs at much higher frequencies, so that at double these frequencies or higher the waves are so short that the tube acts as a large body and no second peak is noticed. Each of the readings quoted in Table XI is the average of a number, the mean variations of which were small.

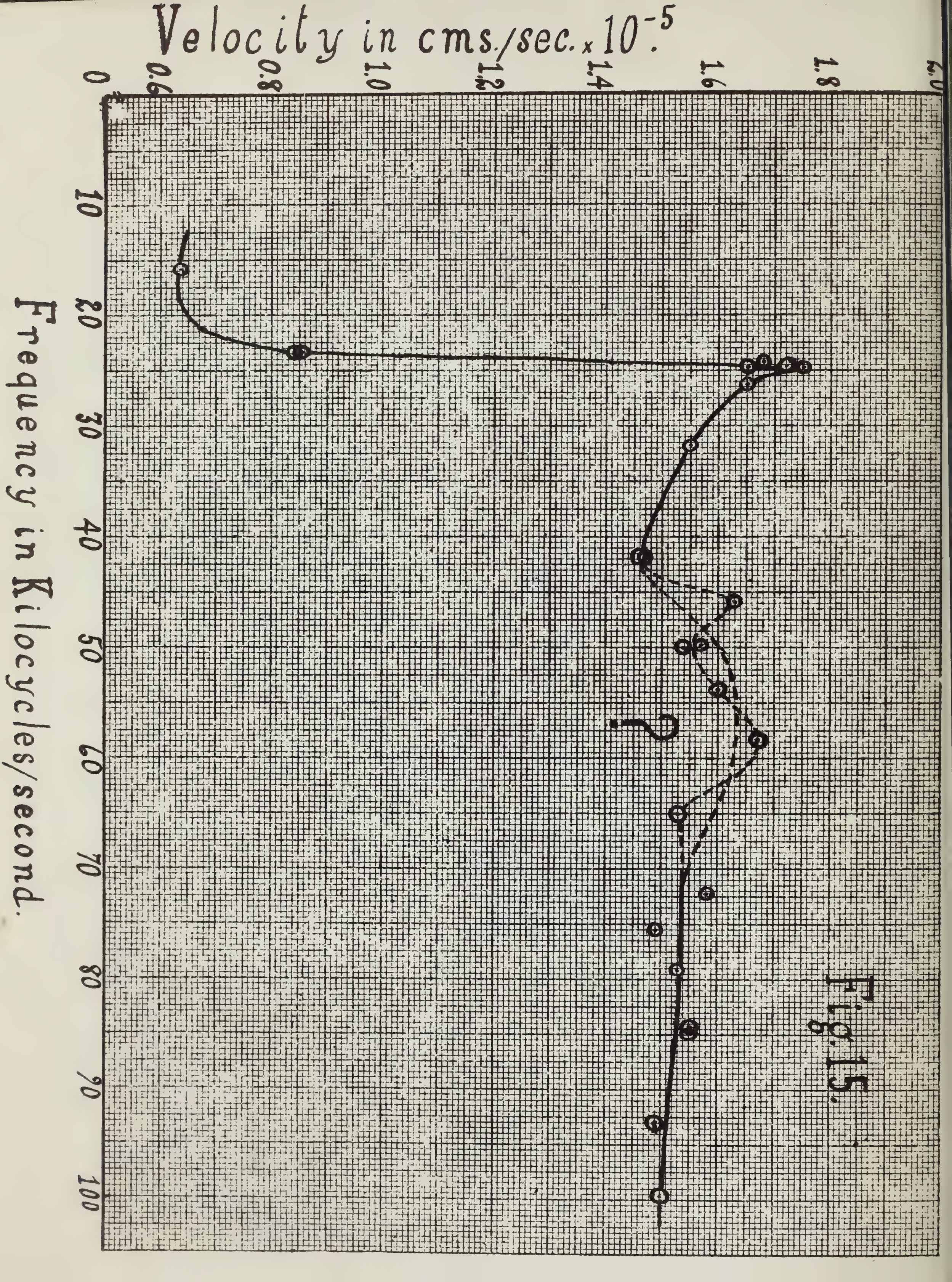

Velocity in cms./sec.x 10^{-5}
0
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
10
20
30
40
50
60
70
80
90
100
Frequency in Kilocycles/second.
?

Fig. 15.

TABLE XI

Length of tube - 47.0 cms. Internal diameter - 7.9 cms.
Wall thickness - 0.2 cms.

No. of nodes observed.	Frequency in cycles/sec.	Wave length in cms.	Velocity in cms./sec.
9	31,750	4.87	1.55 x 10^5
5	24,610	6.98	1.72 "
5	26,140	6.30	1.65 "
4	15,900	4.00	0.636 "
3	24,300	6.50	1.58 "
5	24,600	6.07	1.55 "
3	23,450	3.65	0.855 "
4	23,450	3.60	0.845 "
4	84,200	1.84	1.55 "
11	24,750	6.80	1.75 "
12	75,500	3.97	1.49 "
6	41,850	3.52	1.47 "
8	45,900	3.55	1.65 "
8	49,900	3.14	1.57 "
10	58,200	2.87	1.67 "
4	53,800	3.00	1.60 "
9	50,000	3.08	1.54 "
6	41,900	3.48	1.46 "
6	65,000	2.35	1.53 "
8	72,100	2.20	1.58 "
5	85,000	1.82	1.55 "
4	79,100	1.93	1.53 "
4	93,000	1.60	1.49 "
5	99,700	1.50	1.50 "

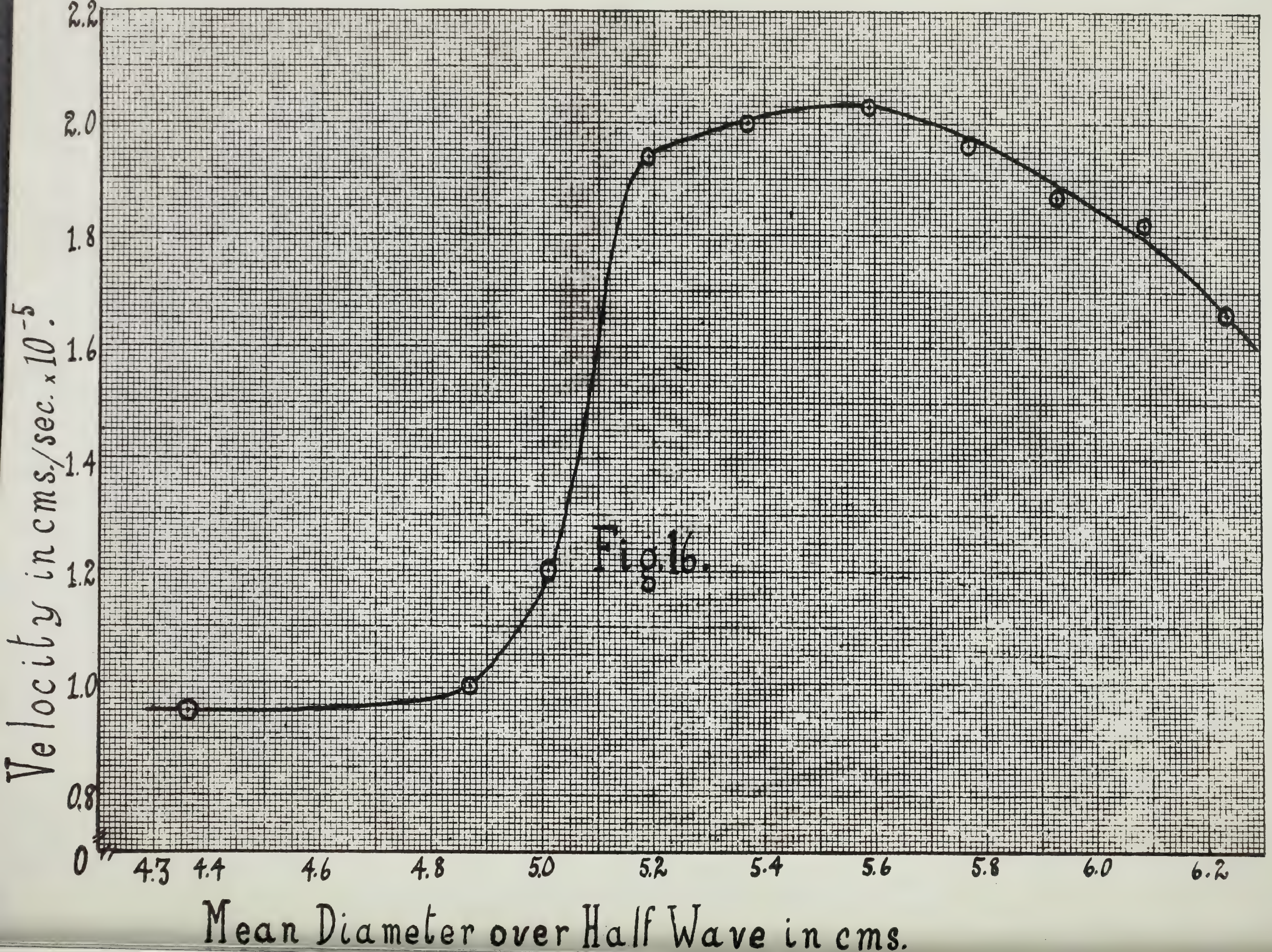

Velocity in cms./sec. x 10^-5.
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0
4.3
4.4
4.6
4.8
5.0
5.2
5.4
5.6
5.8
6.0
6.2
Mean Diameter over Half Wave in cms.

Fig.16.

Influence of Diameter of Column.

Experiments were performed to find whether or not the standing waves would vary in length along a tube whose diameter was not uniform. A conical glass tube (dimensions diameter at large end 6.60 cms., and at small end 4.80 cms., length of tube 39.8 cms. thickness of glass 2.0 mm.) was set up vertically, and a piston oscillator was fitted into its smaller end. It was partially filled with transformer oil, and stationary waves were produced by reflection from a plate in the oil. When the oscillator was actuated at fairly high frequencies no difference could be detected in the length of the standing waves but at a certain lower frequency notable differences in length of stationary waves occurred. In Table III the distance between successive nodes with corresponding calculated velocity is given as well as the mean diameter of the tube corresponding to this particular half wave-length The velocities are plotted against the diameters in Fig.16 The curve rises to a peak at a diameter of about 5.5 cms. The results were anticipated from the fact that the velocity-frequency curve rises to a peak at some frequency which for the same liquid will depend on the diameter of the enclosing tube.

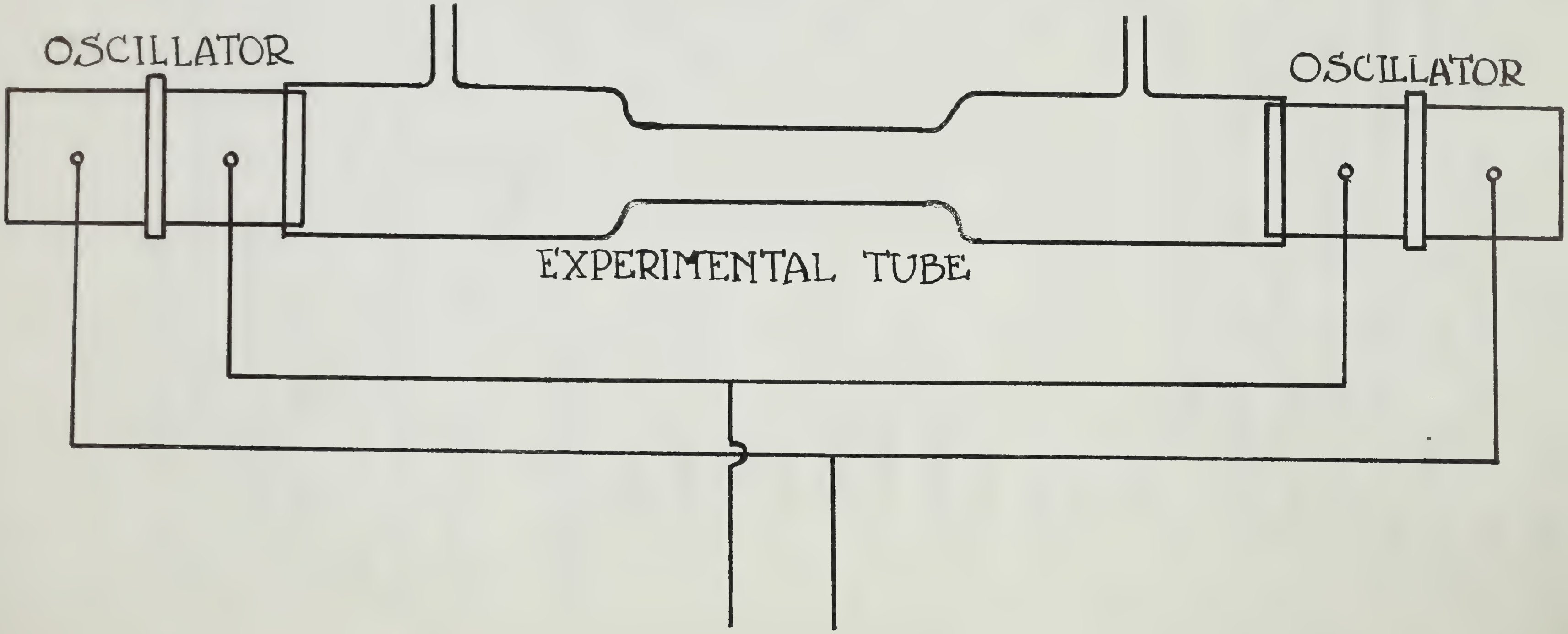
TABLE XII

The frequency of the vibrations was 26,170 cycles per second.

Distance between nodes half wave-length in cms.	Corresponding velocity in cms./sec.	Mean diameter over the half wave in cms.
1.90	0.995×10^5	4.67
2.00	1.05 "	4.36
2.30	1.20 "	5.01
3.70	1.94 "	5.19
3.88	2.00 "	5.37
3.88	2.03 "	5.59
3.75	1.96 "	5.77
3.58	1.87 "	5.93
3.48	1.82 "	6.09
3.10	1.66 "	6.24

A glass tube was blown into the shape shown in Fig.17 to see whether or not at some particular frequency waves of different length would be formed in the sections of the tube of different diameter. The internal diameter of the larger sections was 3.0 cms., that of the smaller 1.7 cms. It was found difficult at most frequencies to make stationary waves form in the smaller section of the tube, but there were three different frequencies at which stationary wave measurements were quite possible. At a frequency of 51,500 cycles per second the wave length in the large sections was 4.40 cms. and in the small section approximately 3 cms; at 81,700 cycles per second the wave length in the large sections was 1.91 cms. and in the small 2.1 cms; at 95,600 the wave length in the large section was 1.67 cms.

Fig. 17.



and in the small 1.86 cms. It is seen from this experimen that the velocity depends upon (1) the frequency and (2) the diameter of the column, and that the frequency can be adjusted so that the velocity is greater in the larger tube than in the smaller or vice versa. The results were, of course, anticipated from the work done previously, and agreed quantitatively with former results.

TABLE XIII

Frequency at the peak in cycles per sec.	Internal diameter of tube in cms.
37,500	3.38
14,000	8.50
72,500	1.86
51,500	2.00
61,000	2.52
40,000	3.20
36,000	3.50
24,800	7.90
26,170	5.60
15,500	9.70
31,000	4.20

The velocities and corresponding peak frequencies were measured using transformer oil as experimental liquid contained in glass tubes of different diameters. In Table XIII the frequencies at which the peaks of velocity occurred and the internal diameters of the tubes are given. In Fig. 18 the frequencies at which the peaks occurred are plotted against the corresponding internal diameters of the tubes.

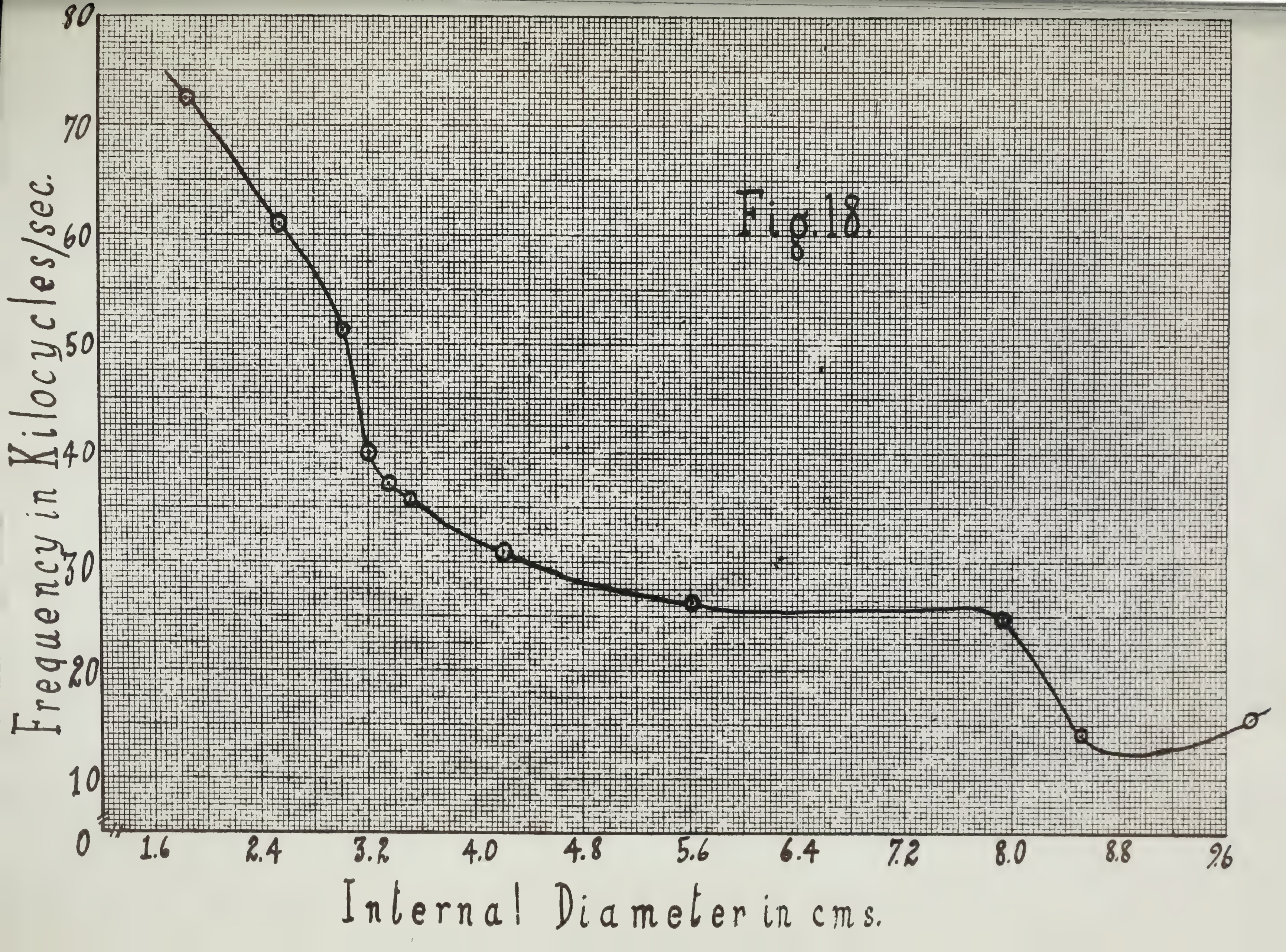


Fig.18.
80
70
60
50
40
30
20
10
0
1.6
2.4
3.2
4.0
4.8
5.6
6.4
7.2
8.0
8.8
9.6
Frequency in Kilocycles/sec.
Internal Diameter in cms.

Velocity in Air.

The velocity of sound in air was measured in several tubes by using a steel reflector with a small piece of mica pasted over the hole in the centre and the stethoscope attached, as in the earlier experiments with liquids. No differences in velocities could be found at different frequencies, and the velocities as measured agreed with the unconfined velocities in air at the same temperatures.

Conclusion

It is evident from these experiments that at certain particular frequencies, which fixed some relation of the diameter of the tube to the wave length of the oscillation in the liquid column enclosed by it, there was a marked selective absorption of energy which caused all the usual characteristics of an energy absorption band. Wave-velocity was greatly lowered immediately on the lower side and greatly raised immediately on the higher side of the critical frequency. Similar would be the conditions of amplitude, particle velocity and harmonic pressure in the waves. No doubt it is at frequencies far removed from the absorption band, where the wave-length is either very large or very small compared with the diameter of the column that all previous experiments on velocity have been carried out, that is, on the regular and flat portions of the velocity-frequency curves far removed from the frequency of the discontinuity. It is no doubt, in these ranges that theories like the Helmholtz-Kirchoff theory may be applied. At or near the frequency of an absorption band the velocity changes represented by such theories are relatively insignificant.

Evidently the next problem will be to find definitely what exactly is the cause for this sudden selective absorption of energy. The fact that the frequency at the absorption band does not seem to depend on either the length or the wall thickness of the containing tube indicates that it is neither

the longitudinal nor the flectural (lateral) vibrations in the tube walls which cause the phenomenon; and the fact that the critical frequency shifts with change of diameter and with change of liquid indicated that it is in the column of liquid itself that the energy absorption is taking place, though in what manner is not at present known. There is, of course, the possibility that since there is more than one type of vibration taking place in the piezo-electric quartz driving the oscillators which generate the waves, there may be set up in the oscillator some special oscillation which is communicated to the liquid column and which the liquid column selectively responds to and absorbs. Further experiments will be undertaken later to continue the investigation on such points.

The authors wish to acknowledge with thanks the receipt of a grant of money from the National Research Council to assist this and other ultrasonic research.